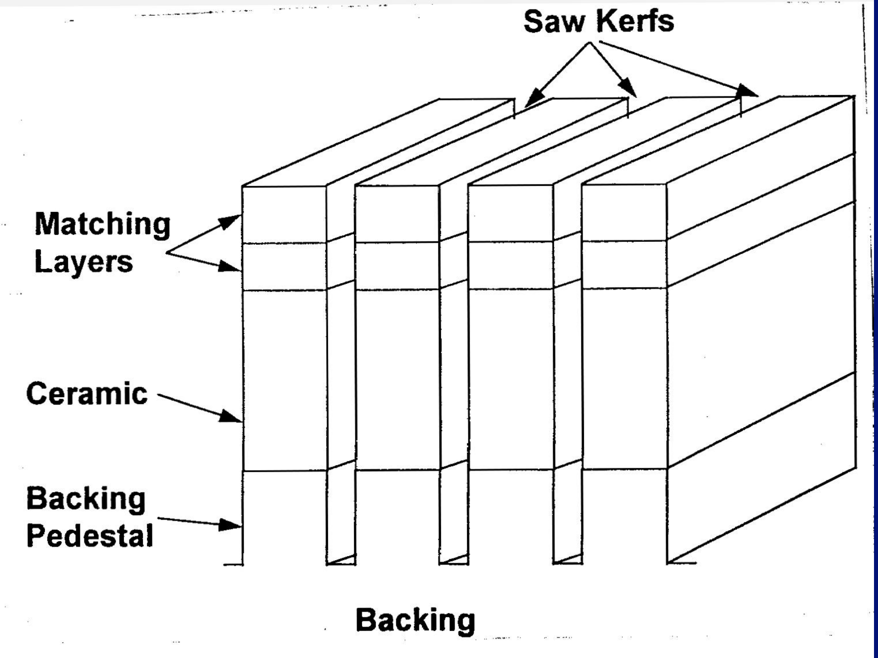
Chapter 4:

Transducers: Generation and Detection of Ultrasound

Ultrasonic Array Transducers

(From www.acuson.com)



Outline

- Piezoelectricity.
- Constitutive relations.
- Wave propagation.
- Generation and detection.
- Equivalent circuits.
- Design considerations.

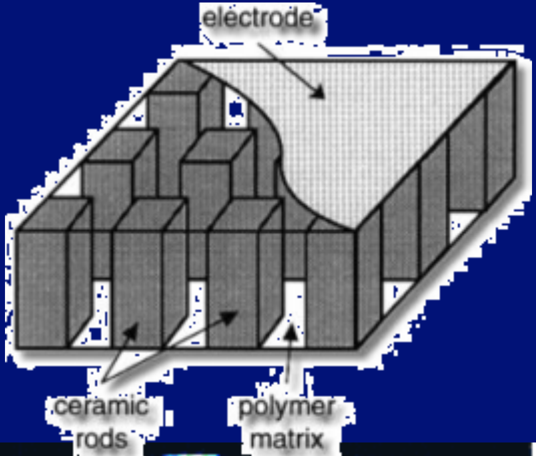
Transducer

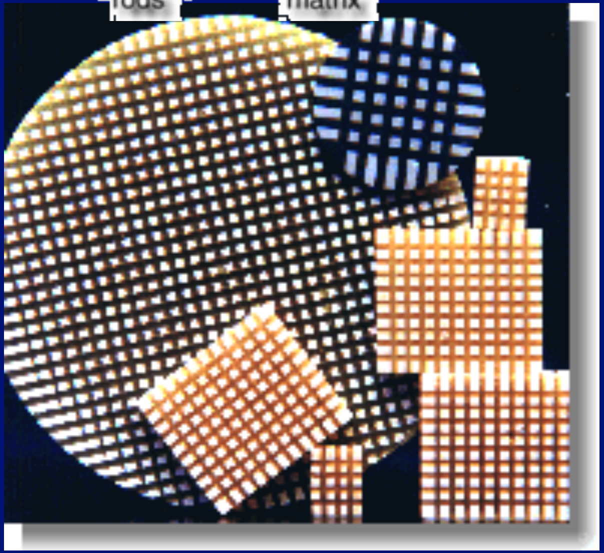
- Energy conversion: electrical ↔ mechanical.
- Generation and detection (speaker and microphone).
- Medical ultrasound: same device in MHz range.
- Piezoelectricity: electrical polarization ↔ mechanical strain.
- PZT, PVDF and composite materials are commonly used.

Piezoelectric Materials

Ceramic

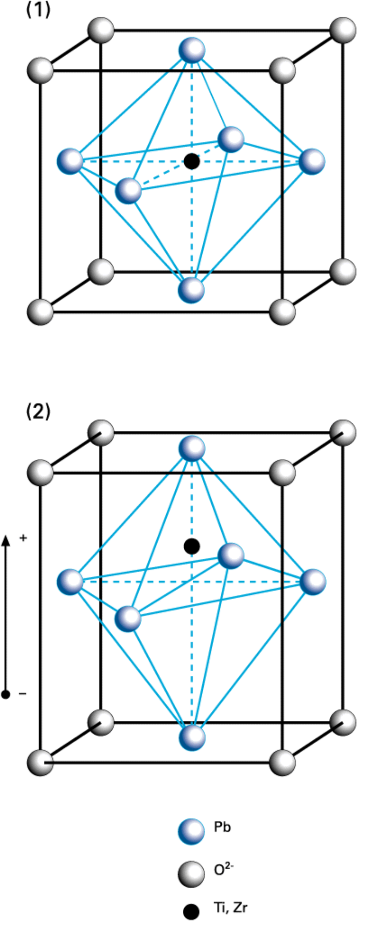
Composite



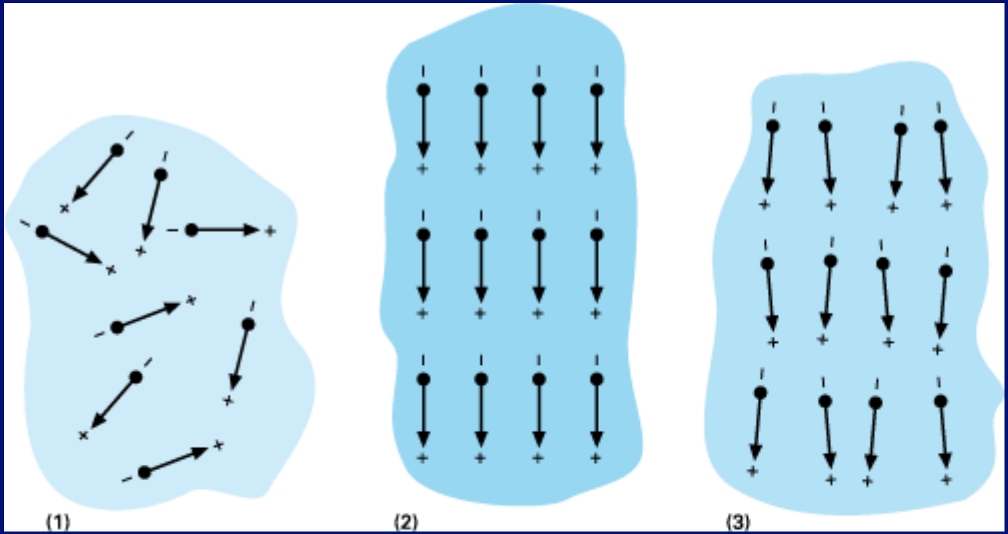


Piezoelectricity

Anisotropy

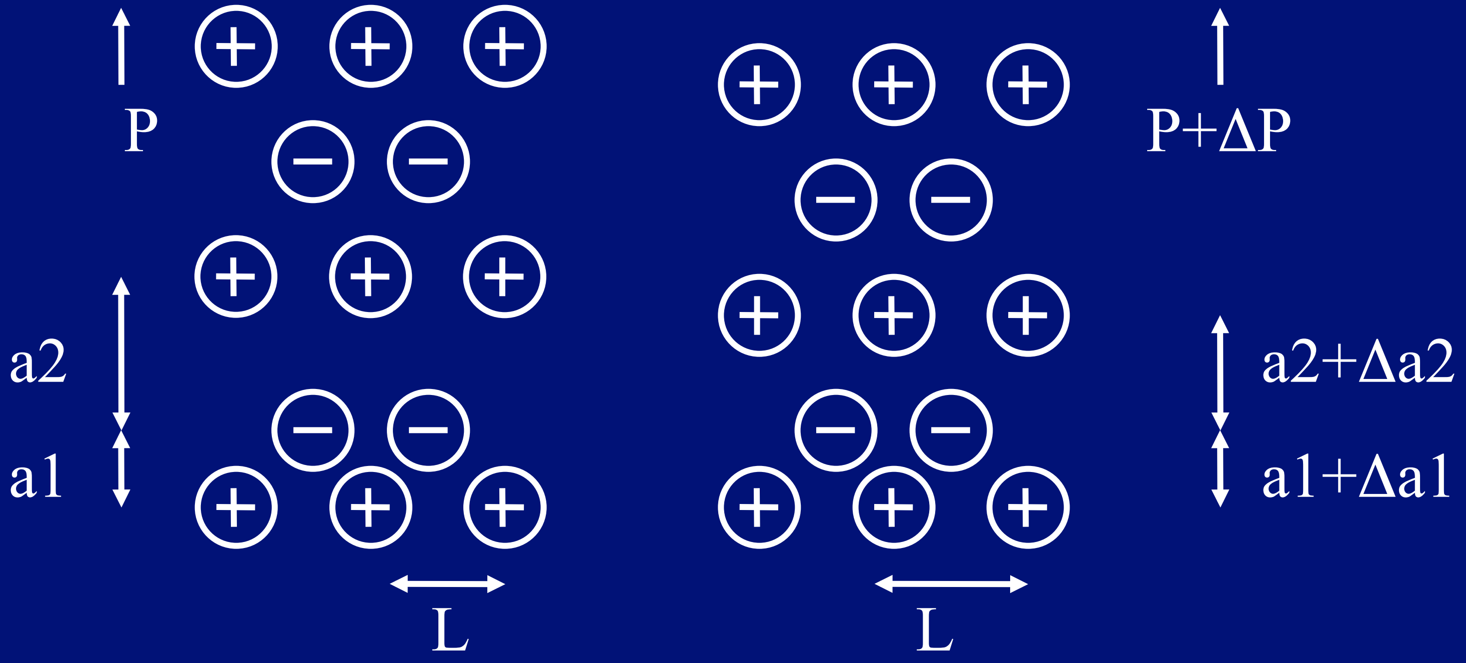


Poling



Curie temperature: $320^0 – 370^0$C.

Piezoelectricity



$$P \equiv \frac{\text{dipole strength of unit cell}}{\text{volume of unit cell}} = \frac{q(a2-a1)}{L^2(a2+a1)}$$

Piezoelectric stress constant

$$\Delta P \approx PS \equiv eS \Rightarrow D = \varepsilon E + \Delta P = \varepsilon E + eS$$

Permittivity

Constitutive Relations

$$T_1 = -\frac{qE}{L^2},\ T_2 = \frac{qE}{L^2}$$

$$T_E = \frac{a1\,T_1 + a2\,T_2}{a1 + a2} = eE$$

Applied stress

$$T + T_E = c_E S \qquad (T_{piezo} = T_{nonpiezo} + T_{elect})$$

Internal stress

⇒

$$D = \varepsilon E + eS$$

$$T = c_E S - eE$$

Wave Propagation

- Newton's law + Constitutive relations.
- Bulk modulus → C_E.

$$\frac{\partial^2 w(z,t)}{\partial t^2} = (c_E/\rho)\frac{\partial T(z,t)/c_E}{\partial z} = (c_E/\rho)\left\{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial\left(eE(z,t)/c_E\right)}{\partial z}\right\}$$

$$\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\rho}{c_E}\frac{\partial^2 w(z,t)}{\partial t^2} = \frac{1}{c_E}\frac{\partial\left(eE(z,t)\right)}{\partial z}$$

Wave Propagation

- Open circuit (constant D) vs. Short circuit (constant E):

$$\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\rho}{c_E}\frac{\partial^2 w(z,t)}{\partial t^2} = \frac{e}{\varepsilon c_E}\frac{\partial D(z,t)}{\partial z} - \frac{e^2}{\varepsilon c_E}\frac{\partial^2 w(z,t)}{\partial z^2} = -\frac{e^2}{\varepsilon c_E}\frac{\partial^2 w(z,t)}{\partial z^2}$$

$$\frac{\partial^2 w(z,t)}{\partial z^2} - \left(\frac{\rho}{c_E\left(1 + e^2/\varepsilon c_E\right)}\right)\frac{\partial^2 w(z,t)}{\partial t^2} = 0$$

$$c_{velocity}^{nonpiezo} \equiv \sqrt{\frac{B}{\rho}} \qquad c_{velocity}^{piezo} = c_{velocity}^{nonpiezo}\left(1 + \frac{e^2}{\varepsilon c_E}\right)^{1/2}$$

$$c_D = c_E\left(1 + \frac{e^2}{\varepsilon c_E}\right) \qquad k^2 \equiv \frac{e^2}{\varepsilon c_E}$$

(Electromechanical coupling coefficient)

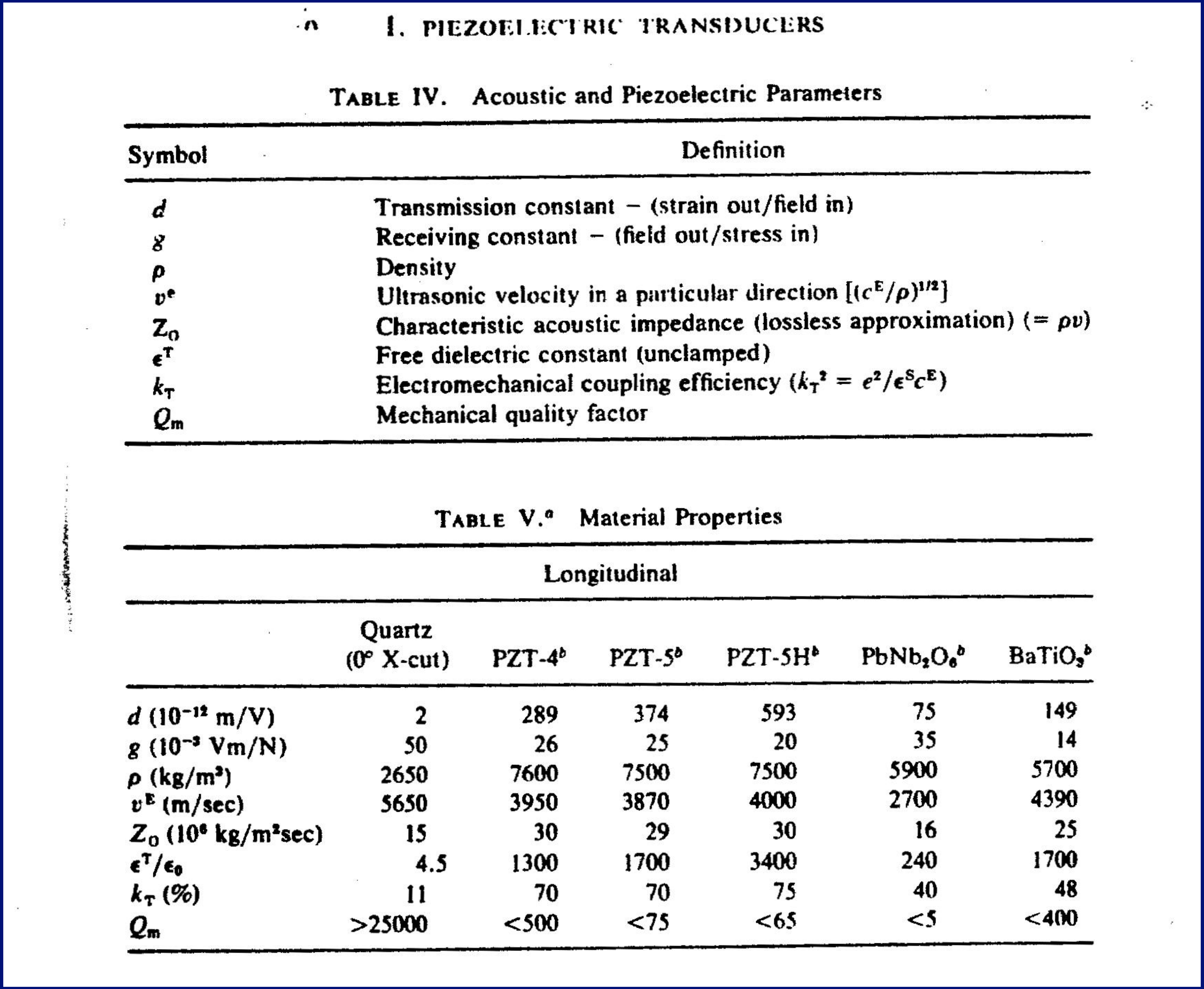


TABLE IV. Acoustic and Piezoelectric Parameters

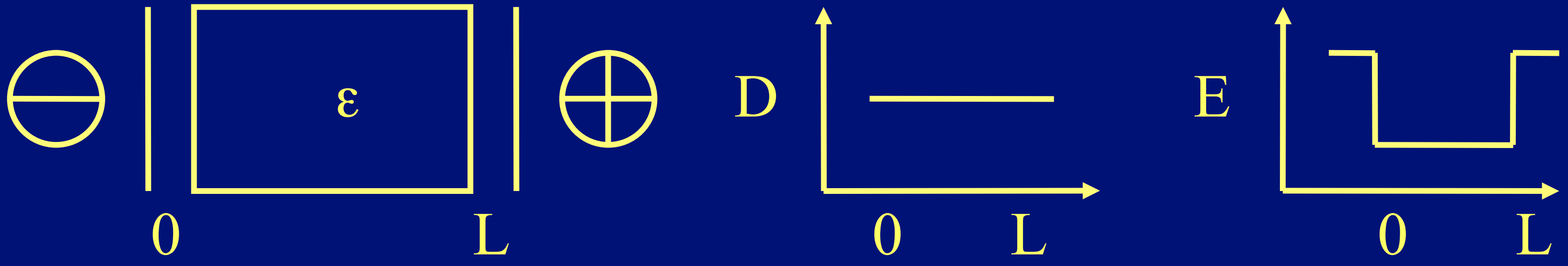
Symbol	Definition
d	Transmission constant – (strain out/field in)
g	Receiving constant – (field out/stress in)
ρ	Density
v^e	Ultrasonic velocity in a particular direction $[(c^E/\rho)^{1/2}]$
Z_0	Characteristic acoustic impedance (lossless approximation) ($= \rho v$)
ϵ^T	Free dielectric constant (unclamped)
k_T	Electromechanical coupling efficiency ($k_T{}^2 = e^2/\epsilon^S c^E$)
Q_m	Mechanical quality factor

TABLE V.[a] Material Properties

	Longitudinal					
	Quartz (0° X-cut)	PZT-4[b]	PZT-5[b]	PZT-5H[b]	$PbNb_2O_6$[b]	$BaTiO_3$[b]
d (10^{-12} m/V)	2	289	374	593	75	149
g (10^{-3} Vm/N)	50	26	25	20	35	14
ρ (kg/m^3)	2650	7600	7500	7500	5900	5700
v^E (m/sec)	5650	3950	3870	4000	2700	4390
Z_0 (10^6 kg/m^{2}sec)	15	30	29	30	16	25
ϵ^T/ϵ_0	4.5	1300	1700	3400	240	1700
k_T (%)	11	70	70	75	40	48
Q_m	>25000	<500	<75	<65	<5	<400

Generation and Detection

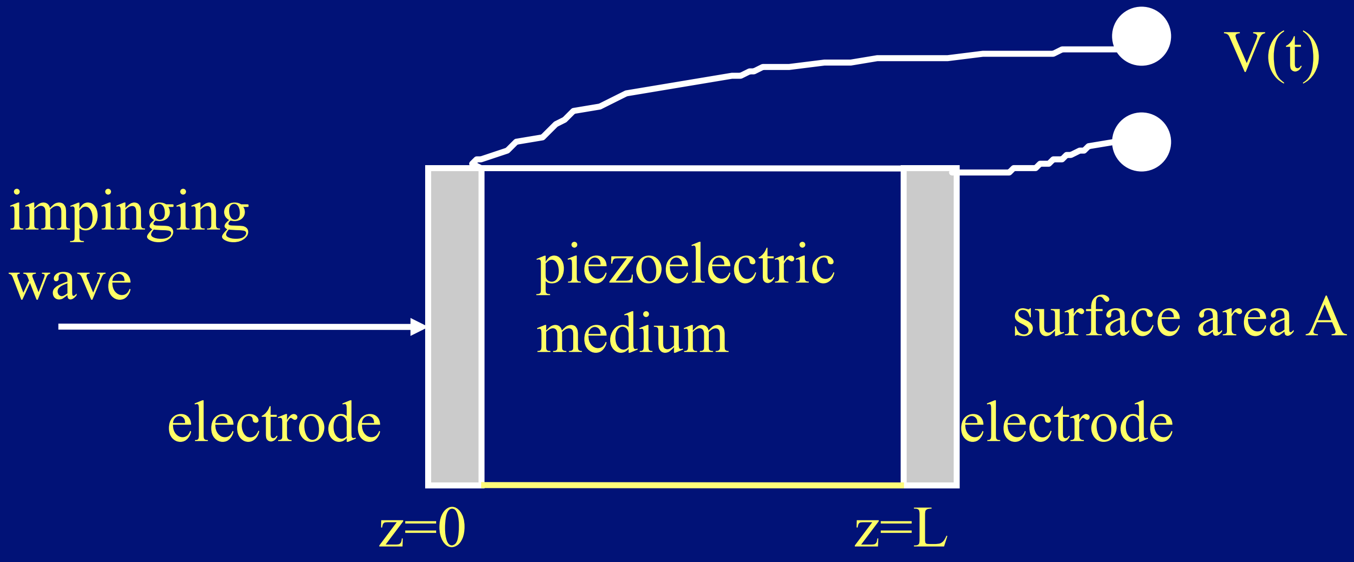
Generation of Ultrasound



- The surfaces of the piezoelectric material are the predominant sources for the generation of ultrasound.

Detection of Ultrasound

- Reciprocal to generation.



$$V(t) = \int_0^L E(z,t)\,dz$$

Detection of Ultrasound

$$V(t) = -\int_0^L \frac{e}{\varepsilon} S(z,t)\,dz + \int_0^L \frac{D(z,t)}{\varepsilon}\,dz$$

$$D(z,t) = \frac{q(t)}{A}$$

$$V(t) = -\frac{e}{\varepsilon}\int_0^L \frac{\partial w(z,t)}{\partial z}\,dz + \frac{q(t)}{\varepsilon A/L}$$

$$\equiv -\frac{e}{\varepsilon}\left(w(L,t) - w(0,t)\right) + \frac{q(t)}{C_0}$$

$$C_0 \equiv \frac{\varepsilon A}{L}$$

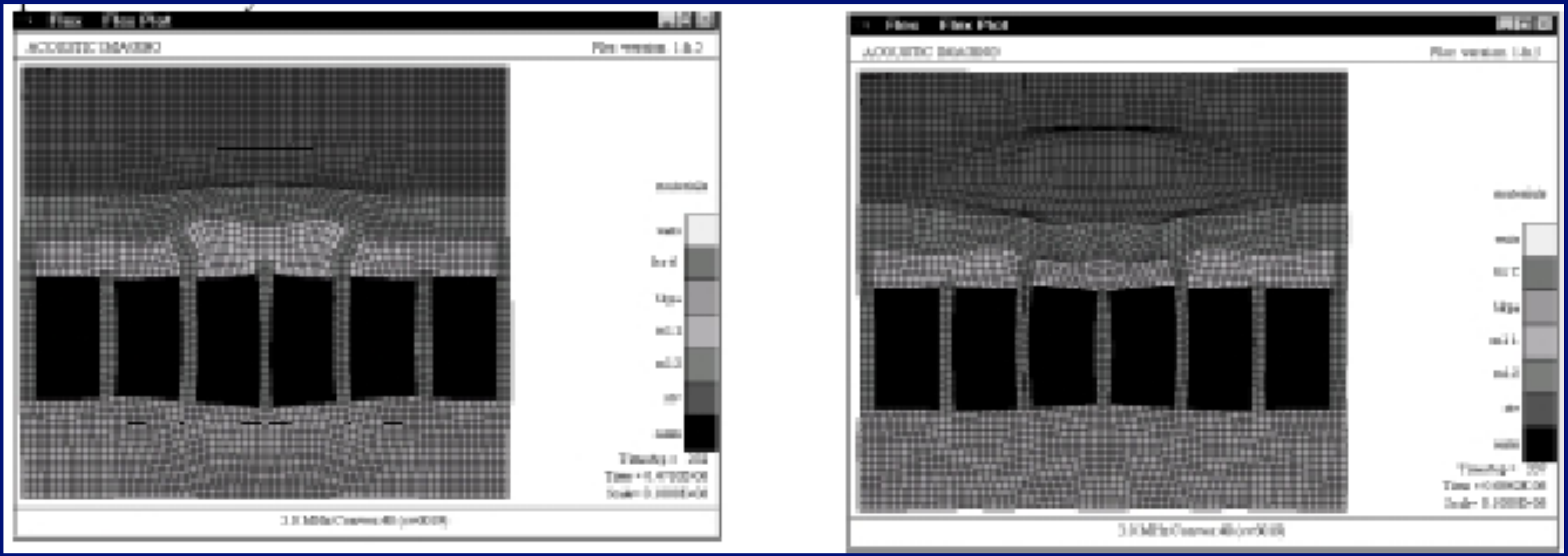
Detection of Ultrasound

- When open circuit:

$$w(L,t)-w(0,t)$$

- Half-wavelength is the most efficient.
- Full-wavelength has no net effect.

Expansion and Contraction

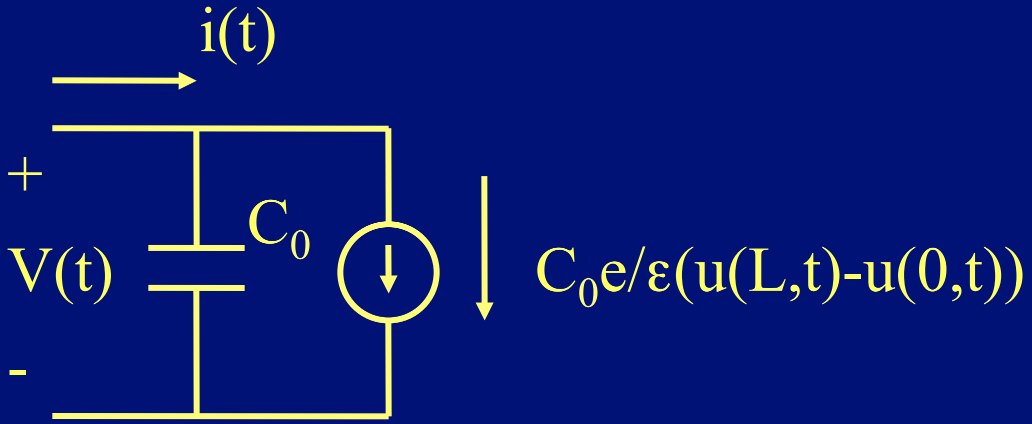


Equivalent Circuits

Electromechanical Coupling

$$V(t) = -\frac{e}{\varepsilon}\Big(w(L,t) - w(0,t)\Big) + \frac{q(t)}{C_0}$$

$$i(t) = C_0\frac{\partial V(t)}{\partial t} + C_0\frac{e}{\varepsilon}\Big(u(L,t) - u(0,t)\Big)$$

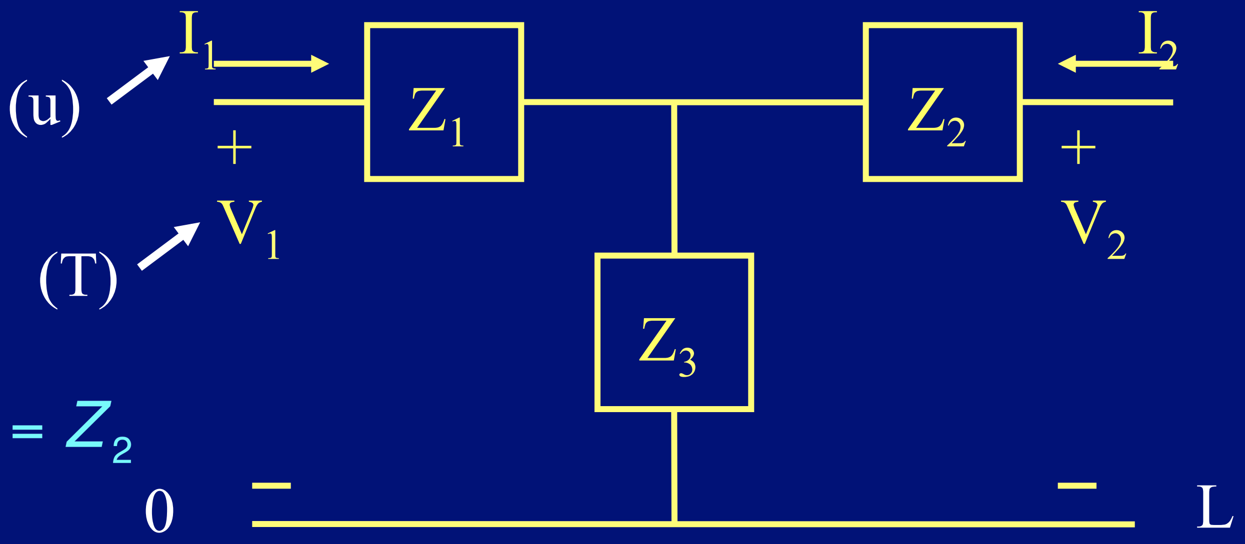


Transmission Line Model

$$T(z,t)_{piezo} = T(z,t)_{nonpiezo} + \frac{e \times q(t)}{\varepsilon \times A}$$

$$V(z,\omega) = V_1(\omega)e^{-j\omega z/c} + V_2(\omega)e^{j\omega z/c}$$

$$I(z,\omega) = \frac{1}{Z_0}\left(V_1(\omega)e^{-j\omega z/c} - V_2(\omega)e^{j\omega z/c}\right)$$



$Z_1 = Z_2$

Equivalent Circuits

$$Z_3 = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{V(L,\omega)}{I(0,\omega)}\bigg|_{I(L,\omega)=0}$$

$$Z_1 + Z_3 = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$I_2 = 0 \quad \Rightarrow$$

$$Z_0 I(L,\omega) = 0 = V_1(\omega)e^{-j\omega L/c} - V_2(\omega)e^{j\omega L/c}$$

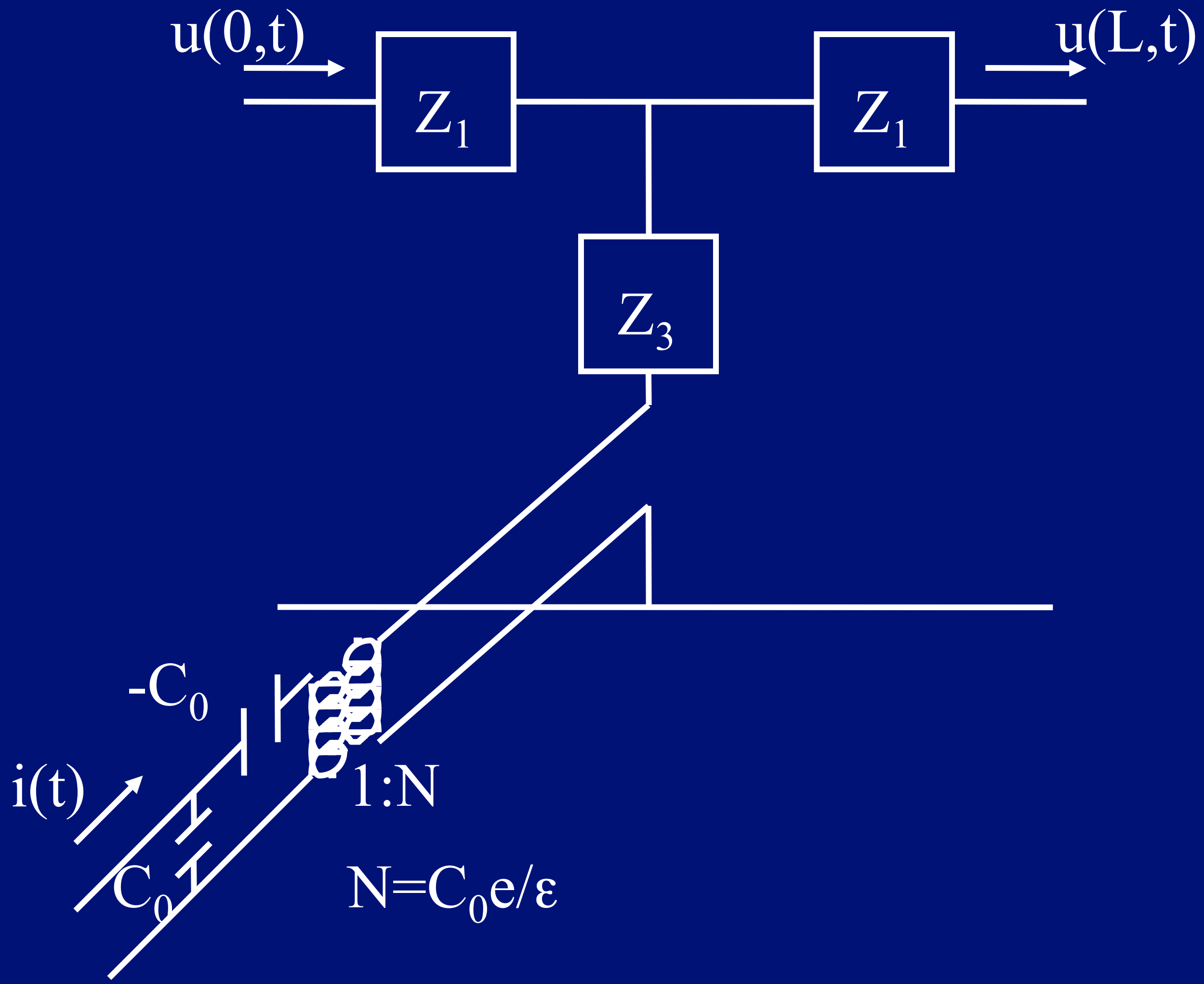
$$V_2(\omega) = V_1(\omega)e^{-2j\omega L/c}$$

Equivalent Circuits

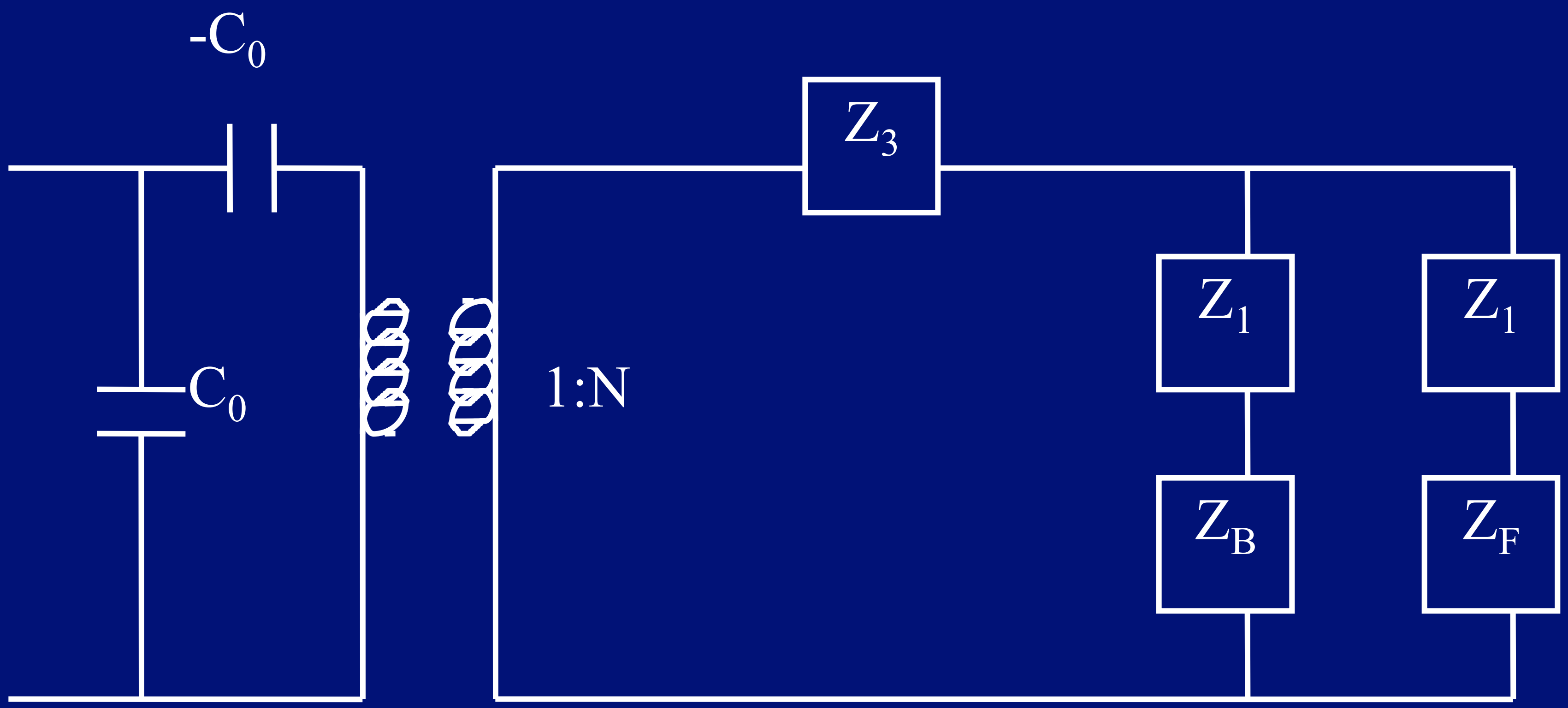
$$Z_3 = Z_0 \frac{V_1(\omega)\left(e^{-j\omega L/c} + e^{-j\omega L/c}\right)}{V_1(\omega)\left(1 - e^{-2j\omega L/c}\right)} = \frac{Z_0}{j\sin 2\pi L/\lambda} = -jZ_0 \operatorname{cosec} 2\pi L/\lambda$$

$$Z_1 = Z_2 = \left.\frac{V(0,\omega)}{I(0,\omega)}\right|_{I_2=0} - Z_3 = Z_0 \frac{\cos 2\pi L/\lambda - 1}{j\sin 2\pi L/\lambda} = jZ_0 \tan \pi L/\lambda$$

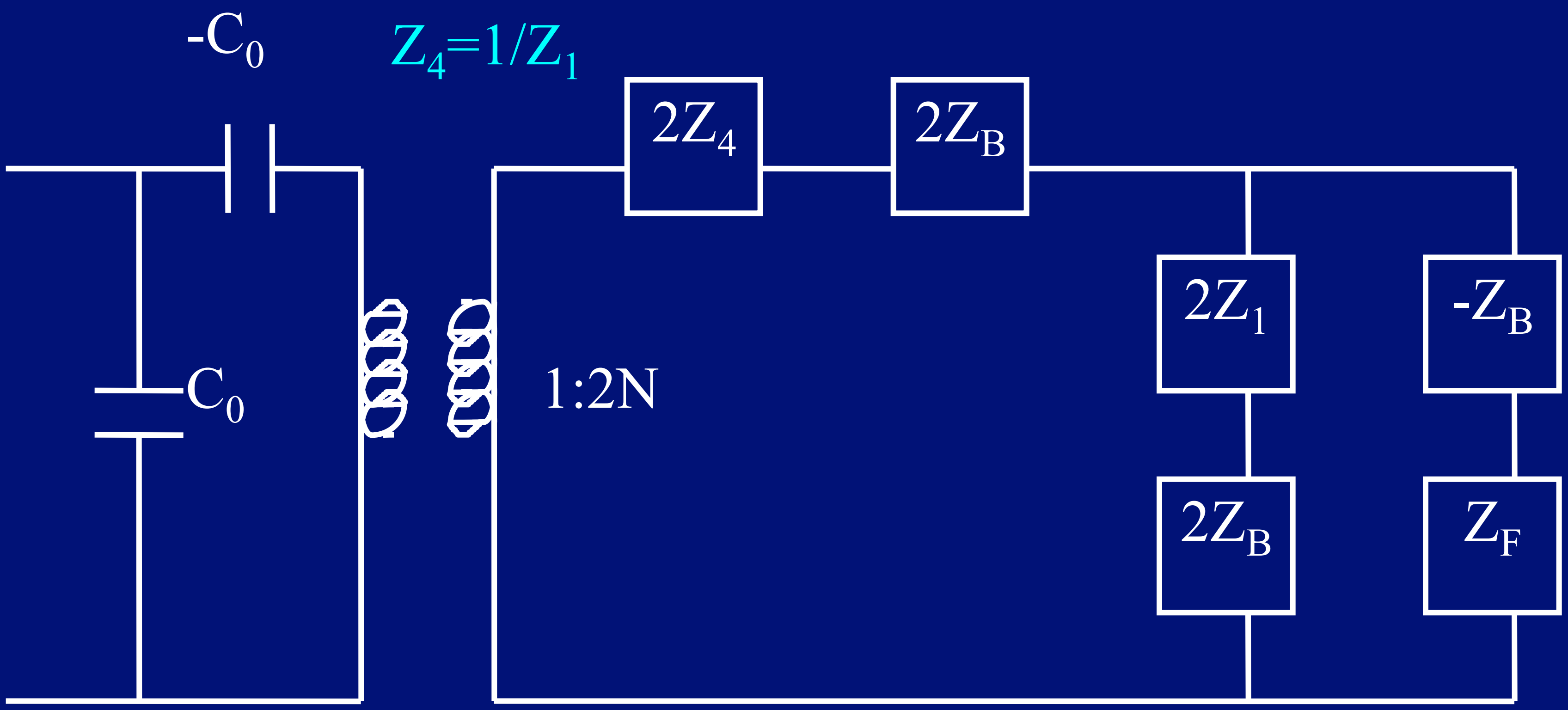
Equivalent Circuits



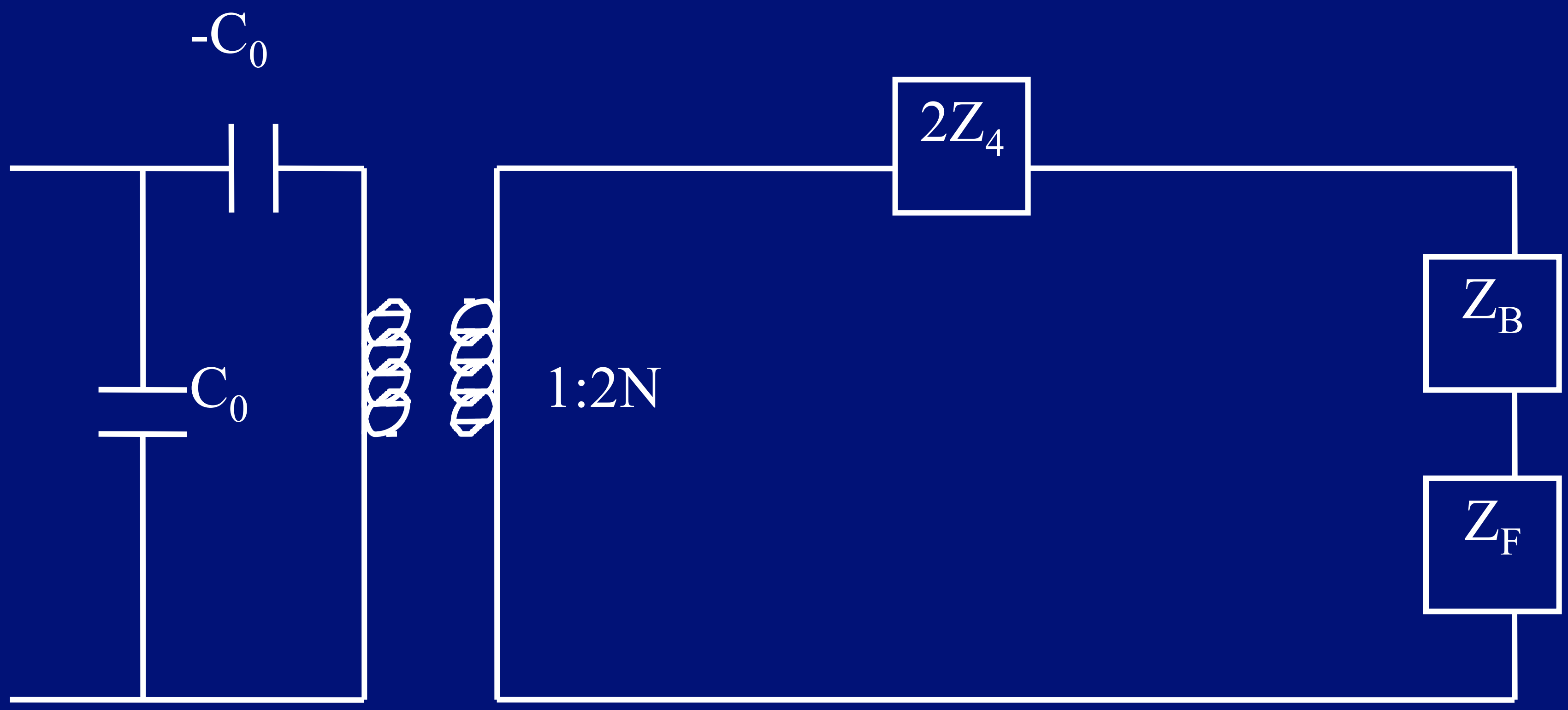
Equivalent Circuits



Equivalent Circuits



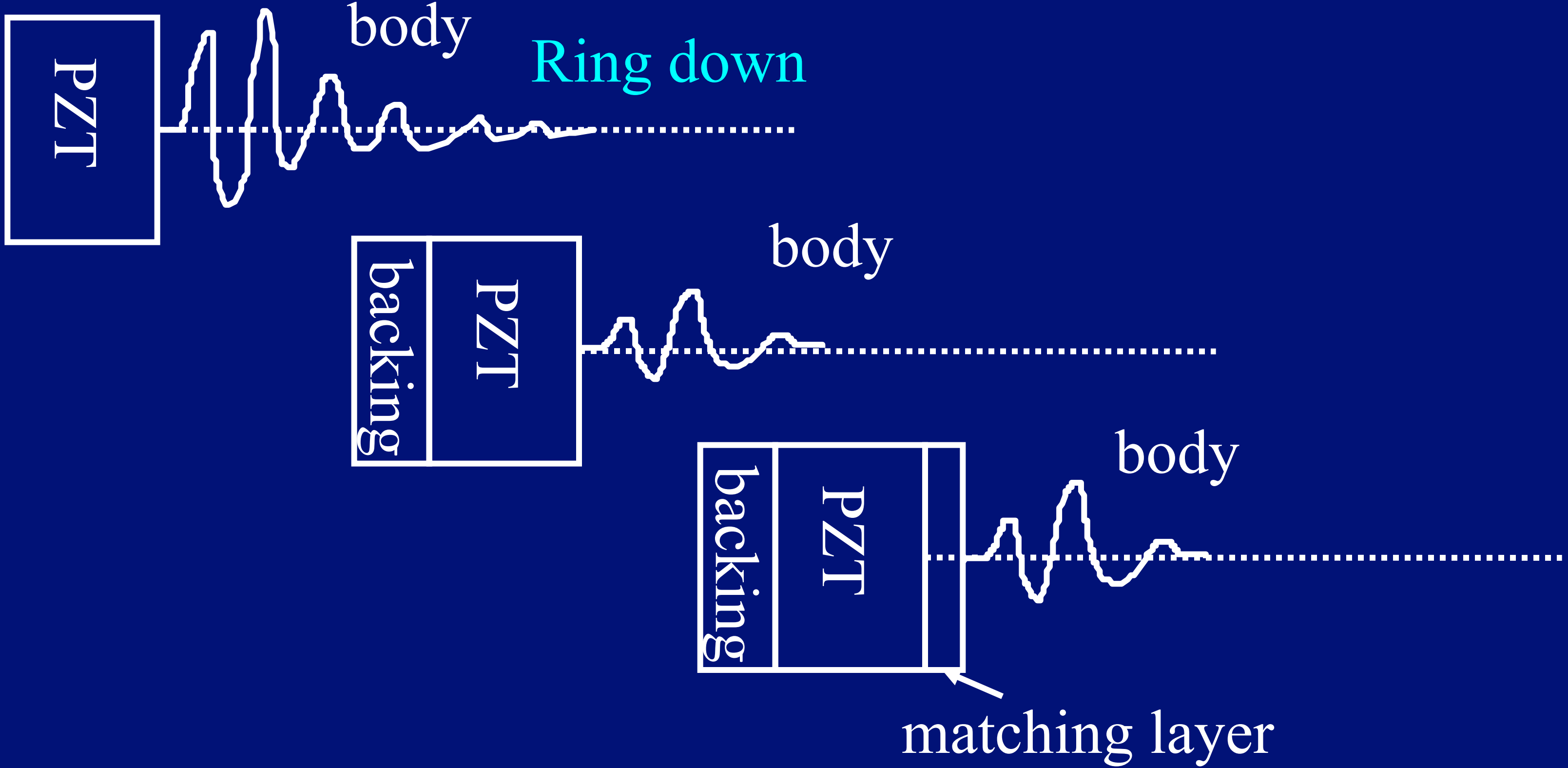
Equivalent Circuits



Near resonance

Design Considerations

- Bandwidth and sensitivity.



Two-Way Insertion Loss

- A measure of the electromechanical efficiency of the transducer.

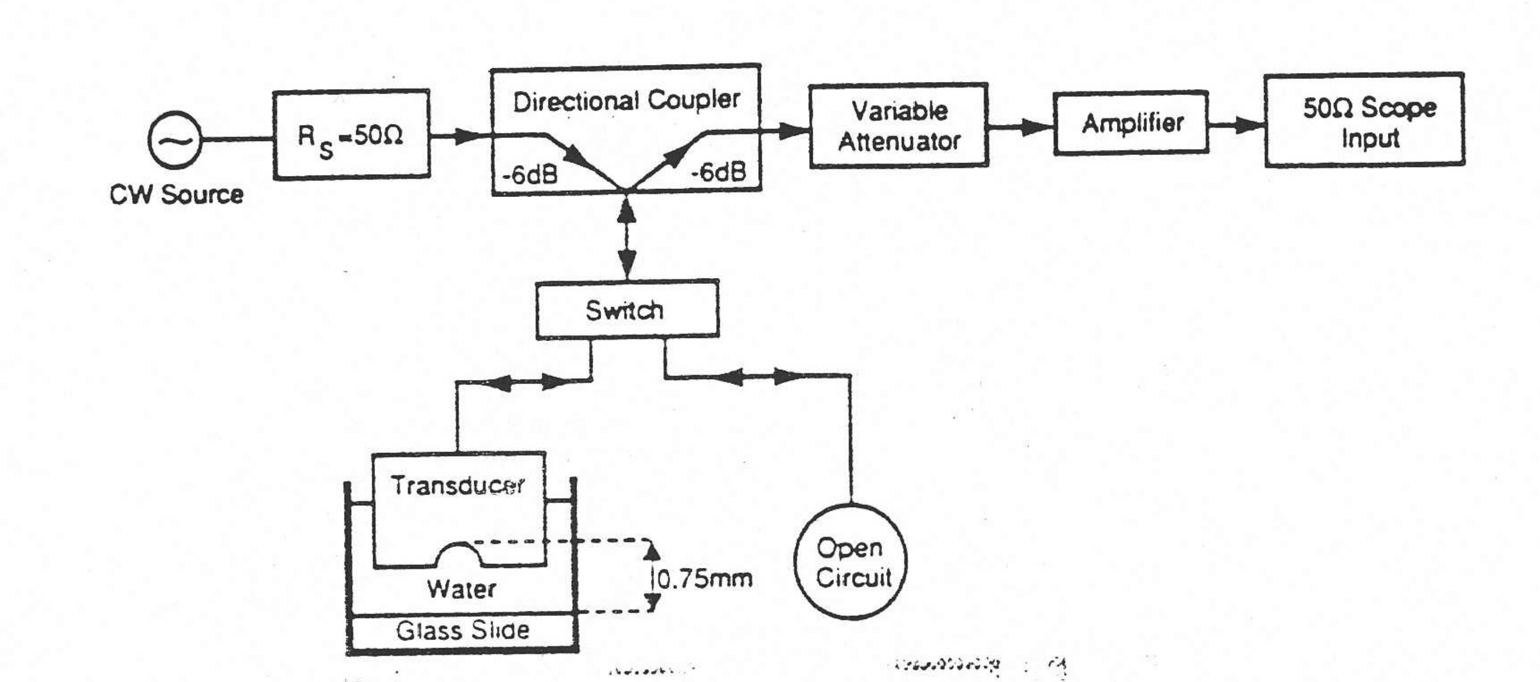
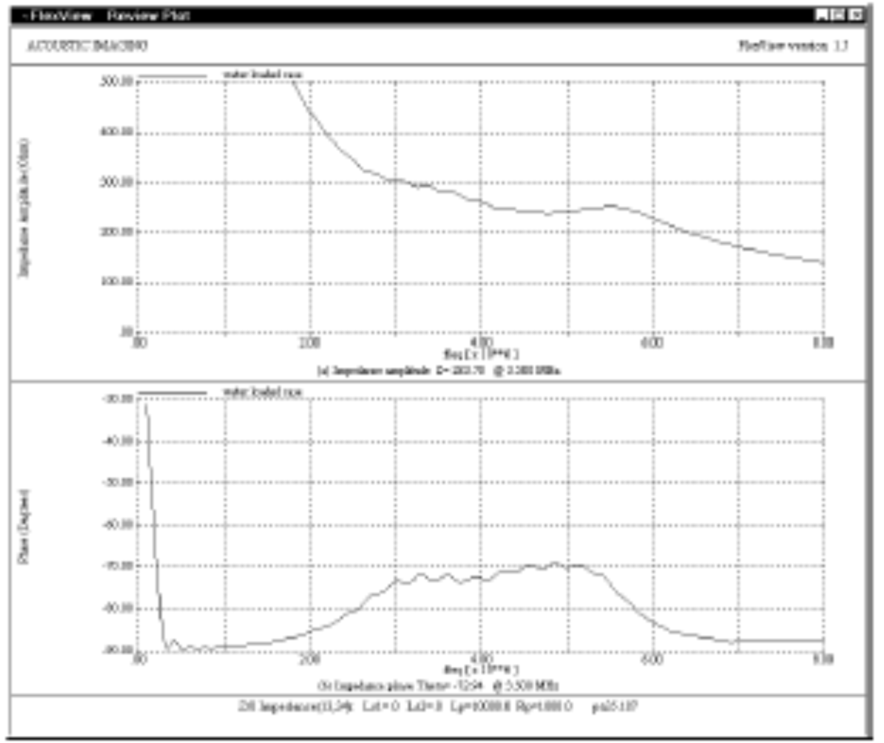


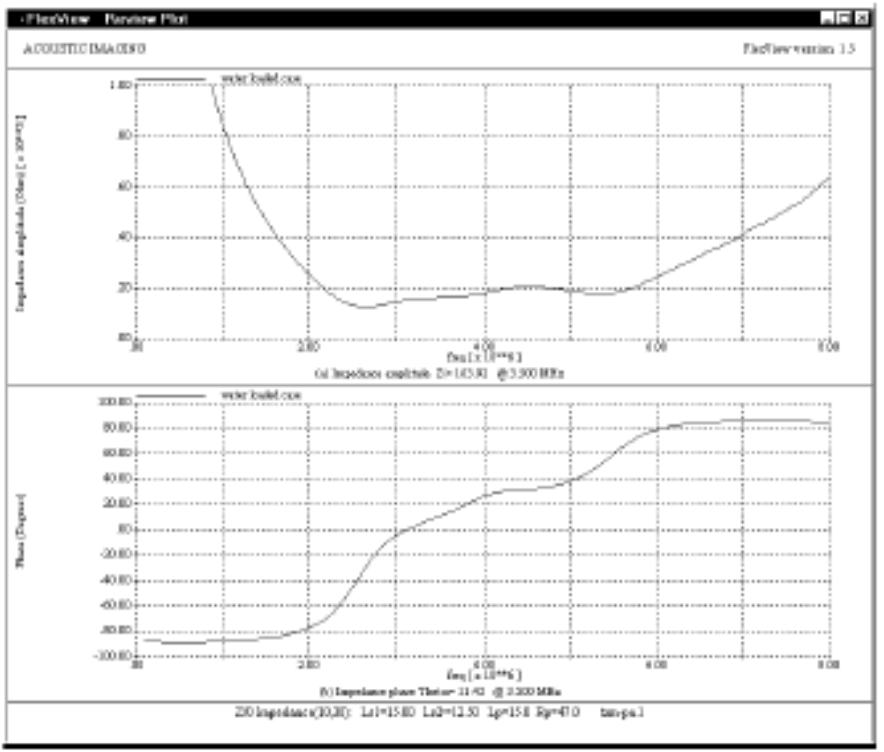
Fig. 6. Apparatus for measurement of transducer insertion loss.

Electrical Tuning

- Inductance vs. capacitance.



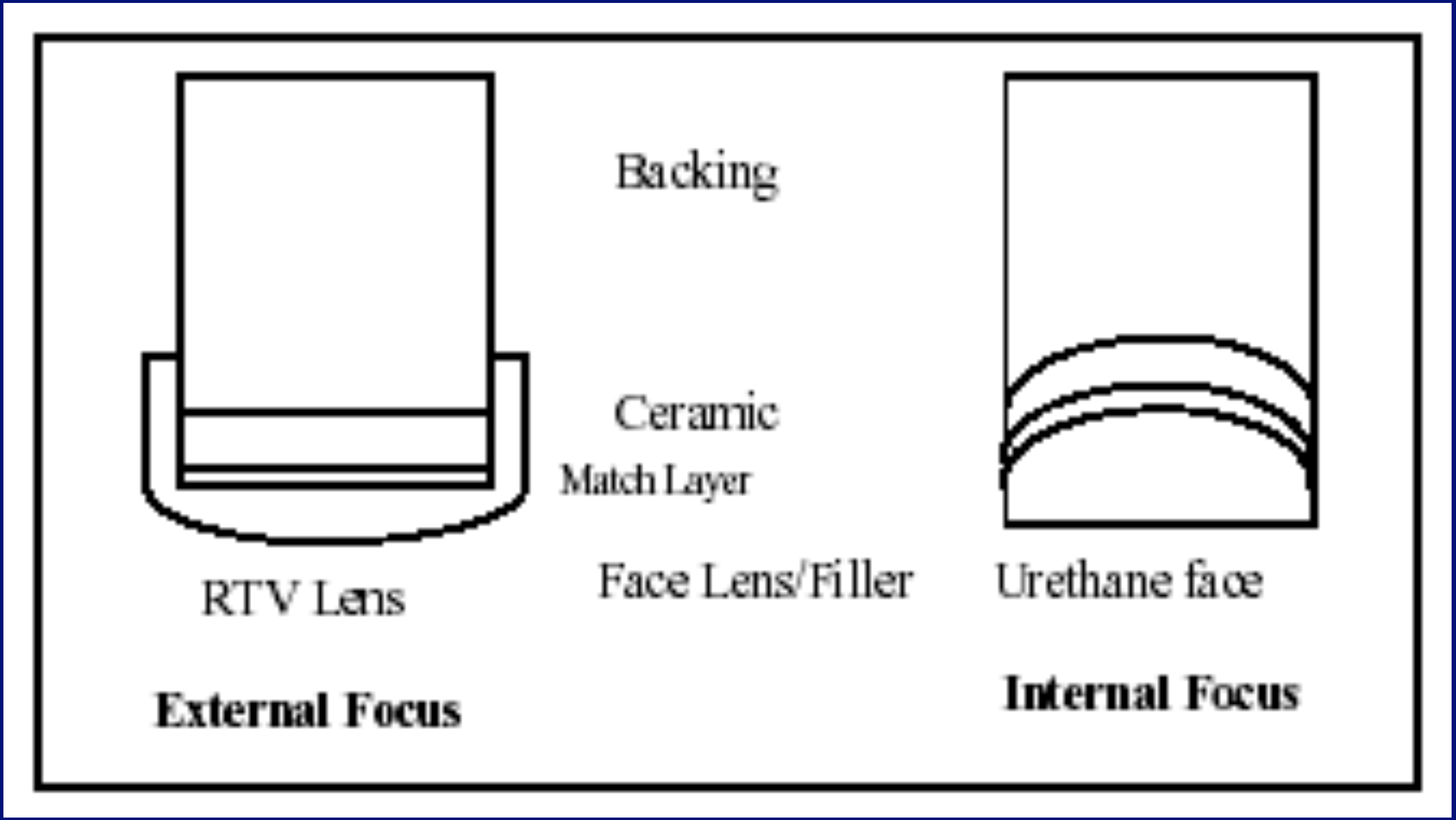
No Tuning



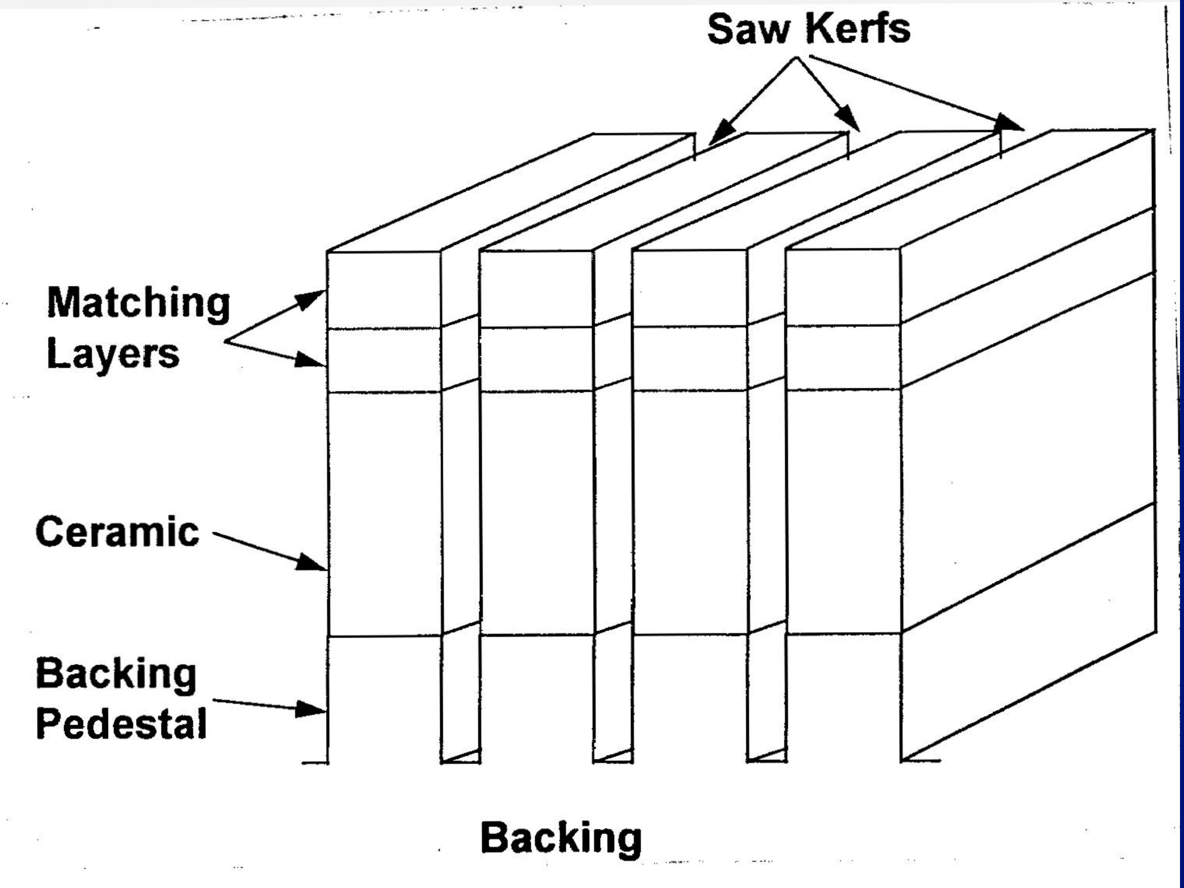
With Tuning

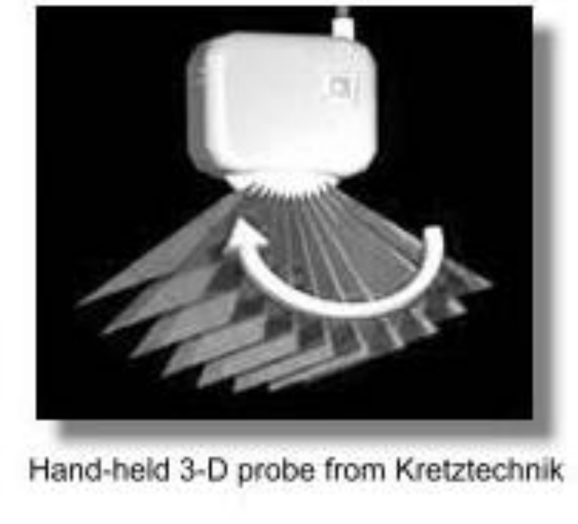
Acoustic Lens

- Fixed geometric elevational focusing.

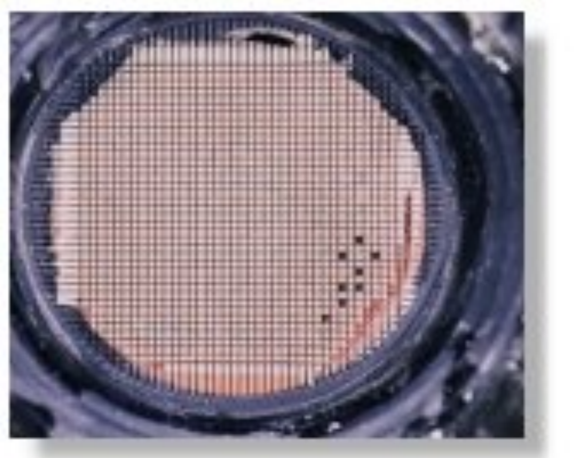


1-D and 2-D Arrays





Hand-held 3-D probe from Kretztechnik



2-D matrix-array at Duke

Issues with 2-D Arrays

- SNR
  - Signal vs. noise
- Interconnection
- System complexity
- Real-time requirements